



AN EOQ MODEL WITH STOCK DEPENDENT DEMAND, VARIABLE HOLDING COST AND DETERIORATION WITH PARTIAL BACKLOGGING IN INFLATIONARY ENVIRONMENT

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Abstract: In this paper, we have developed an order level inventory system for deteriorating items. Deterioration is assumed to be time dependent. The demand is linear function of current inventory level. Shortages are allowed and partially backlogged. Backlogging rate is variable which depends upon the duration of waiting time up to the arrival of next lot. Effect of inflation is also considered in the model. Numerical example is used to illustrate the model. Sensitivity analysis is also performed to study the effect of change in various parameters on the behavior of the model.

Keywords: Inventory, Deterioration, Partial backlogging, Stock dependent demand, Inflation

1. Introduction

Inventory is an important part of our manufacturing, distribution and retail infrastructure where demand plays an important role in choosing the best inventory policy. Researchers were engaged to develop the inventory models assuming the demand of the items to be constant, linearly increasing or decreasing, exponential increasing or decreasing with time. Inventory models with time-dependent demand were studied by Dave (1981) and Maiti et al. (2009).

For certain types of items, particularly consumer goods, the demand rate may be influenced by the inventory levels, that is, the demand may go up or down with the on-hand inventory level. Gupta and Vrat (1986) first developed a model for consumption environment to minimize the cost with the assumption that stock-dependent consumption rate is a function of the initial stock level. Sarker et al. (1997) have been done for order-level lot size inventory model with inventory-level dependent demand and deterioration. Datta and Paul (2001) analyzed an inventory system where the demand rate is influenced by both displayed stock level and selling price. Balkhi and Benkherouf (2004) presented an inventory model for deteriorating items with stock-dependent and time-varying demand rates over a finite planning horizon. Later, it has been realized that the demand of certain items such as newly launched fashion items, garments, cosmetics, automobiles etc, the demand increases with time as they are launched into the market and after some time, it becomes constant. In order to consider demand of such types, the concept of ramp-type demand is introduced. Mandal and Pal (1998) have developed inventory models with ramp type demand rate for deteriorating items. Panda et al. (2008) have developed optimal replenishment policy for perishable seasonal products taking ramp-type time dependent demand rate. Avinadav et al. (2013) have developed considered demand function sensitive to price and time. Models for seasonal products with ramp-type time-dependent demand are discussed by Wang and Huang (2014). Wu et al. (2016) proposed a model for two inventory systems with trapezoidal type demand. San Jose et al. (2017) have developed an inventory model with demand

dependent on both time and price.

Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage and loss of utility or loss of marginal value of a commodity that reduces usefulness from original ones. Blood, fish, fruits and vegetables, alcohol, gasoline, radioactive chemicals, medicines, etc., lose their utility with respect to time. In this case, a discount price policy is implemented by the suppliers of these products to promote sales. Thus, decay or deterioration of physical goods in stock is a very realistic feature. Modelers felt the need to take this factor into consideration. Various types of order-level inventory models for items deteriorating at a constant rate were discussed by Shah and Jaiswal (1977) and Dave (1986). As time progressed, several researchers developed inventory models with variable deterioration rate. In this connection, researchers may consult the work by Covert and Philip (1973), Chakrabarti et al. (1998), Jalan et al. (1996) and Dye (2004), who have used Weibull distribution for representing deterioration rate. Manna and Chaudhuri (2006) have developed an EOQ model with ramp type demand rate and time dependent deterioration rate. An inventory system with Markovian demands, phase type distributions for perishability and replenishment is developed by Chakravarthy (2011). San-José et al. (2014) have studied inventory system with partial backlogging and mixture of dispatching policies. Teng et al. (2016) studied inventory lot-size policies for deteriorating items with expiration dates and advance payments. Chan et al. (2017) have developed an production-inventory model with deterioration during delivery.

When shortage for a product occurs, some customers will go away, while some would like to wait for backlogging after the next replenishment. But the willingness is diminishing with the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Thus practically, all shortages are not backlogged but only some part of shortages is backlogged. This phenomenon is called partial backlogging.

Chang and Dye (1999) developed an inventory model in which the demand rate is a time-continuous function and items deteriorate at a constant rate with partial backlogging rate which is the reciprocal of a linear function of the waiting time. Papachristos and Skouri (2000) developed an EOQ inventory model with time-dependent partial backlogging. Teng et al. (2003) then extended the backlogged demand to any decreasing function of the waiting time up to the next replenishment. The related analysis on inventory systems with partial backlogging have been performed by Teng and Yang (2004), Dye et al. (2006) etc. Singh and Singh (2007, 2009) studied inventory model with partial backlogging considering quadratic demand and power demand. San-Jose et al. (2015) have studied partial backlogging with non linear holding cost.

In most of the models mentioned above, the inflation and time value of money were disregarded. It has happened mostly because of the belief that the inflation and the time value of money would not influence the inventory policy to any significant degree. However, most of the countries have suffered from large-scale inflation and sharp decline in the purchasing power of money last several years. As a result, while determining the optimal inventory policy, the effects of inflation and time value of money cannot be ignored. The pioneer research in this direction was Buzacott (1975), who developed an EOQ model with inflation subject to different types of pricing policies. Vrat and Padmanabhan (1990) developed an inventory model under a constant inflation rate for initial stock-dependent consumption rate. Datta and Pal (1991) developed a model with linear time-dependent demand rate and shortages to investigate the effects of inflation and time value of money on ordering policy over a finite time horizon. Ray and Chaudhuri (1997) and Wee and Law (2001) all have investigated the effects of inflation, time

value of money and deterioration on inventory models. Dey et al.(2008) formulated two storage inventory problem over finite horizon under inflation. Mondal et al. (2013) developed a production repairing inventory model under inflation. Yadav et al. (2015) studied retailer's optimal policy under inflation. Tiwari et al. (2016) have taken impact of trade credit and inflation into consideration. Alikar et al. (2017) have studied a bi-objective multi-period allocation problem with time value of money and inflation considerations.

In this paper, an effort has been made to analyse an EOQ model for time-dependent deteriorating items assuming the demand rate to be a linear function of present inventory level. Shortages are allowed and partially backlogged. Backlogging rate is variable which depends upon the duration of waiting time up to the arrival of next lot. Effect of inflation is also considered in the model. Numerical example is used to illustrate the model. Sensitivity analysis is also performed to study the effect of change in various parameters on the behavior of the model.

2. Assumptions and Notations:

To develop an inventory model with variable demand and partial backlogging, the following notations and assumptions are used:

- i) Replenishment is instantaneous and lead time is zero
- ii) $c_1 e^{kt}$ is the inventory holding cost per unit per unit of time, where c_1 and k are constants.
- iii) c_2 is the deterioration cost per unit per unit of time.
- iv) c_3 is the shortage cost per unit per unit of time.
- v) c_4 is the unit cost of lost sales.
- vi) Ordering cost is c' .
- vii) $Q(t)$ be the inventory level at time t .
- viii) Demand rate is dependent on present inventory level $Q(t)$ and defined by $f(t) = aQ(t) + b$ where a & b are constants.
- ix) Unsatisfied demand is backlogged at a rate $e^{-\lambda t}$, where t is the time up to next replenishment and λ is a positive constant.
- x) R is the total cost per production cycle and T is the time for each cycle.
- xi) A variable fraction $\theta(t) = \alpha t$, ($0 < \alpha \ll 1, t \geq 0$) is the deterioration rate.
- xii) A constant r represents the inflation rate.

3. Formulation and solution of the model:

At the start of the cycle, the inventory level reaches its maximum S units of item at time $t=0$. During the time interval $[0, t_1]$, the inventory depletes due mainly to demand and partly to deterioration. At time $t < t_1$, the inventory level depletes and at t_1 , the inventory level is zero and all the demand hereafter (i.e. $T - t_1$) is partially backlogged. As the demand is dependent on present stock, it varies with time up to time t_1 and become constant thereafter. The deterioration

rate is described by an increasing function of time $\theta(t) = \alpha t$.

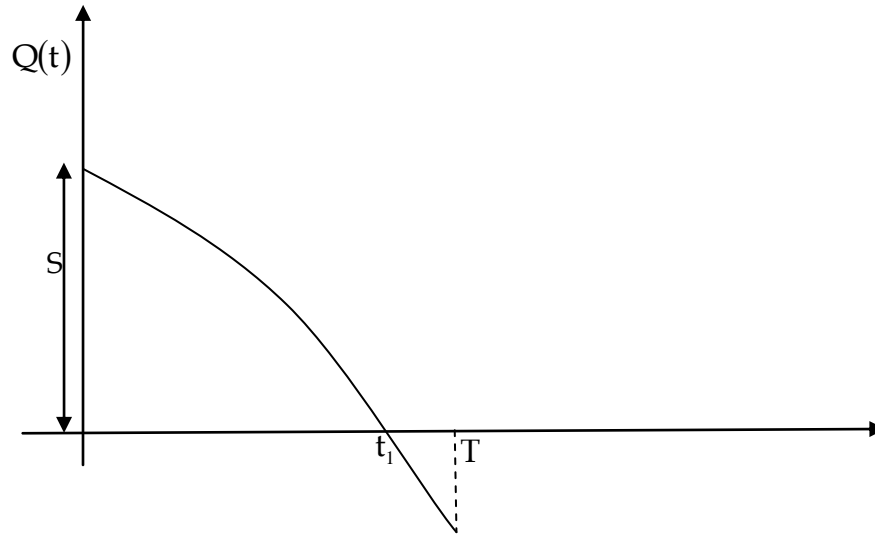


Figure 1

A Graphical representation of the considered inventory system is given in the figure 1 above.

The differential equations governing the instantaneous states of $Q(t)$ in the interval $[0, T]$ are as follows:

$$\frac{dQ}{dt} + \theta(t)Q(t) = -f(t), 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dQ}{dt} = -f(t) e^{-\lambda t}, t_1 \leq t \leq T \quad (2)$$

Conditions are $Q(0) = S, Q(t_1) = 0$

The solutions of equations (1) and (2) are given below:

$$Q(t) = \frac{1}{6}(6S - 6bt - 6aSt + 3abt^2 + 3a^2bt^3) + \frac{1}{6}(-3St^2 + 2bt^3 + \frac{5}{2}abt^4)\alpha, 0 \leq t \leq t_1 \quad (3)$$

$$Q(t) = \frac{be^{-t\lambda}}{\lambda} - \frac{be^{-t_1\lambda}}{\lambda}, t_1 \leq t \leq T \quad (4)$$

The present value of inventory holding cost during the interval $(0, T)$ is given by

$$C_H = c_1 \left[\int_0^{t_1} e^{kt} e^{-rt} Q(t) dt \right]$$

Taking $k - r = m$ and integrating, we get ,

$$C_H = -\frac{c_1(-3a^2bm + abm^2 + bm^3 + am^3S + m^4S + 10aba - 2bma - m^2S\alpha)}{m^5} + \frac{1}{12m^5} c_1 e^{mt_1} (-36a^2bm + 12abm^2 + 12bm^3 + 12am^3S + 12m^4S + 36a^2bm^2t_1 - 12abm^3t_1 - 12bm^4t_1 - 12am^4St_1 - 18a^2bm^3t_1^2 + 6abm^4t_1^2 + 6a^2bm^4t_1^3 + 120aba - 24bma - 12m^2S\alpha - 120abmt_1\alpha + 24bm^2t_1\alpha + 12m^3St_1\alpha + 60abm^2t_1^2\alpha - 12bm^3t_1^2\alpha - 6m^4St_1^2\alpha - 20abm^3t_1^3\alpha + 4bm^4t_1^3\alpha + 5abm^4t_1^4\alpha) \quad (5)$$

The present value of cost due to deterioration of units in the period $(0, T)$ is given by

$$C_D = c_2 \left[\int_0^{t_1} \theta(t) e^{-rt} Q(t) dt \right] \Rightarrow C_D = \frac{c_2(12a^2b + 3abr - 2br^2 - 2ar^2S + r^3S)\alpha}{r^5} - \frac{1}{2r^5} c_2 e^{-rt_1} (24a^2b + 6abr - 4br^2 - 4ar^2S + 2r^3S + 24a^2brt_1 + 6abr^2t_1 - 4br^3t_1 - 4ar^3St_1 + 2r^4St_1 + 12a^2br^2t_1^2 + 3abr^3t_1^2 - 2br^4t_1^2 - 2ar^4St_1^2 + 4a^2br^3t_1^3 + abr^4t_1^3 + a^2br^4t_1^4)\alpha \quad (6)$$

The present value of cost due to shortages in the interval $(0, T)$ is given by

$$C_S = -c_3 \left[\int_{t_1}^T e^{-rt} Q(t) dt \right] \Rightarrow C_S = bc_3 \left[-\frac{e^{-rT-t_1\lambda}}{r\lambda} + \frac{e^{-rt_1-t_1\lambda}}{r\lambda} + \frac{e^{-rT-T\lambda}}{\lambda(r+\lambda)} - \frac{e^{-rt_1-t_1\lambda}}{\lambda(r+\lambda)} \right] \quad (7)$$

The present value of opportunity cost due to lost sales in the interval $(0, T)$ is given by

$$C_O = c_4 \left[\int_{t_1}^T (1 - e^{-\lambda t}) e^{-rt} f(t) dt \right] \Rightarrow C_O = bc_4 \left[\frac{-e^{-rT} + e^{-rt_1}}{r} + \frac{e^{-T(r+\lambda)} - e^{-t_1(r+\lambda)}}{r+\lambda} \right] \quad (8)$$

The total R of present values of costs in the system in the interval $(0, T)$ is given by

$$R = c' + C_H + C_D + C_S + C_O \quad (9)$$

In above relation, c' is constant, while C_H, C_D, C_S & C_O are given by the equations (5) to (8).

The average K of present values of cost in the system in the interval $(0, T)$ is given by

$$K = \frac{R}{T} \quad (10)$$

The optimum values of t_1 and T which minimize K are obtained by using the equations:

$$\frac{\partial K}{\partial t_1} = 0 \text{ and } \frac{\partial K}{\partial T} = 0,$$

Now,

$$\frac{\partial K}{\partial t_1} = 0$$

$$\begin{aligned} \Rightarrow & -\frac{1}{2r^5} c_2 e^{-rt_1} (24a^2br + 6abr^2 - 4br^3 - 4ar^3S + 2r^4S + 24a^2br^2t_1 + 6abr^3t_1 - 4br^4t_1 \\ & - 4ar^4St_1 + 12a^2br^3t_1^2 + 3abr^4t_1^2 + 4a^2br^4t_1^3)\alpha + \frac{1}{2r^4} c_2 e^{-rt_1} (24a^2b + 6abr \\ & - 4br^2 - 4ar^2S + 2r^3S + 24a^2brt_1 + 6abr^2t_1 - 4br^3t_1 - 4ar^3St_1 + 2r^4St_1 + 12a^2br^2t_1^2 \\ & + 3abr^3t_1^2 - 2br^4t_1^2 - 2ar^4St_1^2 + 4a^2br^3t_1^3 + abr^4t_1^3 + a^2br^4t_1^4)\alpha \\ & + \frac{1}{12m^5} c_1 e^{mt_1} (36a^2bm^2 - 12abm^3 - 12bm^4 - 12am^4S - 36a^2bm^3t_1 + 12abm^4t_1 \\ & + 18a^2bm^4t_1^2 - 120abm\alpha + 24bm^2\alpha + 12m^3S\alpha + 120abm^2t_1\alpha - 24bm^3t_1\alpha - 12m^4St_1\alpha \\ & - 60abm^3t_1^2\alpha + 12bm^4t_1^2\alpha + 20abm^4t_1^3\alpha) + \frac{1}{12m^4} c_1 e^{mt_1} (-36a^2bm + 12abm^2 \\ & + 12bm^3 + 12am^3S + 12m^4S + 36a^2bm^2t_1 - 12abm^3t_1 - 12bm^4t_1 - 12am^4St_1 \\ & - 18a^2bm^3t_1^2 + 6abm^4t_1^2 + 6a^2bm^4t_1^3 + 120aba - 24bma - 12m^2S\alpha - 120abmt_1\alpha \\ & + 24bm^2t_1\alpha + 12m^3St_1\alpha + 60abm^2t_1^2\alpha - 12bm^3t_1^2\alpha - 6m^4St_1^2\alpha - 20abm^3t_1^3\alpha \\ & + 4bm^4t_1^3\alpha + 5abm^4t_1^4\alpha) + bc_4 \left[-e^{-rt_1} - \frac{e^{-t_1(r+\lambda)}(-r-\lambda)}{r+\lambda} \right] \\ & + bc_3 \left[\frac{e^{-rT-t_1\lambda}}{r} + \frac{e^{-rt_1-t_1\lambda}(-r-\lambda)}{r\lambda} - \frac{e^{-rt_1-t_1\lambda}(-r-\lambda)}{\lambda(r+\lambda)} \right] = 0 \end{aligned} \tag{11}$$

Also, $\frac{\partial K}{\partial T} = 0$ gives

$$\begin{aligned} \Rightarrow & -c' - \frac{c_2(12a^2b+3abr-2br^2-2ar^2S+r^3S)\alpha}{r^5} + \frac{1}{2r^5} c_2 e^{-rt_1} (24a^2b + 6abr - 4br^2 \\ & - 4ar^2S + 2r^3S + 24a^2brt_1 + 6abr^2t_1 - 4br^3t_1 - 4ar^3St_1 + 2r^4St_1 + 12a^2br^2t_1^2 \\ & + 3abr^3t_1^2 - 2br^4t_1^2 - 2ar^4St_1^2 + 4a^2br^3t_1^3 + abr^4t_1^3 + a^2br^4t_1^4)\alpha \\ & + \frac{c_1(-3a^2bm+abm^2+bm^3+am^3S+m^4S+10aba-2bma-m^2S\alpha)}{m^5} \end{aligned}$$

$$\begin{aligned}
 &-\frac{1}{12m^5}c_1e^{mt_1}(-36a^2bm + 12abm^2 + 12bm^3 + 12am^3S + 12m^4S + 36a^2bm^2t_1 \\
 &-12abm^3t_1 - 12bm^4t_1 - 12am^4St_1 - 18a^2bm^3t_1^2 + 6abm^4t_1^2 + 6a^2bm^4t_1^3 + 120aba \\
 &-24bma - 12m^2S\alpha - 120abmt_1\alpha + 24bm^2t_1\alpha + 12m^3St_1\alpha + 60abm^2t_1^2\alpha - 12bm^3t_1^2\alpha \\
 &-6m^4St_1^2\alpha - 20abm^3t_1^3\alpha + 4bm^4t_1^3\alpha + 5abm^4t_1^4\alpha) \\
 &-bc_4\left[\frac{-e^{-rT}+e^{-rt_1}}{r} + \frac{e^{-T(r+\lambda)}-e^{-t_1(r+\lambda)}}{r+\lambda}\right] - bc_3\left[-\frac{e^{-rT-t_1\lambda}}{r\lambda} + \frac{e^{-rt_1-t_1\lambda}}{r\lambda} + \frac{e^{-rT-T\lambda}}{\lambda(r+\lambda)} - \frac{e^{-rt_1-t_1\lambda}}{\lambda(r+\lambda)}\right] + \\
 &T\left[bc_4\left\{e^{-rT} + \frac{e^{-T(r+\lambda)}(-r-\lambda)}{r+\lambda}\right\} + bc_3\left\{\frac{e^{-rT-t_1\lambda}}{\lambda} + \frac{e^{-rT-T\lambda}(-r-\lambda)}{\lambda(r+\lambda)}\right\}\right] = 0 \tag{12}
 \end{aligned}$$

The above two equations can be solved simultaneously to obtain numerical solution for the values of t_1 and T .

4. **Numerical Example:** To illustrate the model numerically, we use the following parameter values:

$$c' = 500, c_1 = 4, c_2 = 3, c_3 = 6, c_4 = 8, \alpha = 0.008, \\
 \lambda = 0.06, k = -0.3, a = 0.1, b = 200, S = 2000$$

Applying the subroutine FindRoot in Mathematica 8, we obtain the optimal solution for t_1 and T as follows:

$$t_1 = 18.2586, \quad T = 20.7229$$

Also, the optimal value of K for these parameters is 898.23

5. **Sensitivity Analysis:**

Sensitivity analysis is performed by changing (increasing and decreasing) most of the parameters by 10%, 30% and 50%, and taking one parameter at a time, keeping the remaining parameters at their original values. Thus following table is formed:

Table 1

Changing Parameter	% Change	t_1	T	Average Cost	% change in average cost
c'	+50	18.3457	20.9397	910.2	1.33
	+30	18.3109	20.8525	905.5	0.81
	+10	18.2760	20.7659	900.6	0.26
	-10	18.2412	20.6801	895.8	-0.27
	-30	18.2064	20.5948	891.0	-0.80
	-50	18.1715	20.5102	886.1	-1.35
c_1	+50	20.9153	29.6831	1178.9	31.25
	+30	19.7051	25.1446	1080.3	20.27

	+10	18.6920	21.9012	962.5	7.16
	-10	17.8380	19.6765	830.7	-7.52
	-30	16.9906	17.8132	685.6	-23.67
	-50	16.0749	16.0998	526.5	-41.38
c_2	+50	16.9117	19.2788	951.2	5.90
	+30	17.3604	19.7528	932.4	3.80
	+10	17.9204	20.3544	910.6	1.38
	-10	18.6491	21.1533	884.6	-1.52
	-30	19.6569	22.2884	852.1	-5.14
	-50	21.1978	24.1011	808.3	-10.01
c_3	+50	18.1640	19.6916	906.5	0.92
	+30	18.1899	19.9888	904.0	0.64
	+10	18.2296	20.4214	900.5	0.25
	-10	18.2983	21.1180	895.4	-0.32
	-30	18.4508	22.4878	886.9	-1.26
	-50	19.0709	27.3207	867.5	-3.42
c_4	+50	18.0897	18.9246	918.9	2.30
	+30	18.1433	19.5927	912.9	1.63
	+10	18.2150	20.3267	903.9	0.63
	-10	18.3079	21.1418	891.9	-0.70
	-30	18.4257	22.0604	876.9	-2.37
	-50	18.5750	23.1168	858.9	-4.38
α	+50	16.7909	19.1580	957.3	6.58
	+30	17.2891	19.6855	936.5	4.26
	+10	17.8976	20.3347	912.1	1.54
	-10	18.6702	21.1686	882.9	-1.71
	-30	19.7098	22.3108	846.4	-5.77
	-50	21.2487	24.0569	798.0	-11.16
λ	+20	18.3779	21.3451	898.25	0.00
	+15	18.3420	21.1555	898.4	0.02
	+10	18.3108	20.9920	898.4	0.02
	-10	18.2158	20.5091	897.8	-0.05
	-15	18.1970	20.4176	897.6	-0.07
	-20	18.1795	20.3344	897.2	-0.11
k	+20	18.0942	19.9618	824.8	-8.17
	+15	18.1463	20.1424	840.9	-6.38
	+10	18.1931	20.3304	858.3	-4.45
	-10	18.2505	21.1120	947.5	5.49
	-15	18.1954	21.2816	976.9	8.76
	-20	18.0846	21.4069	1010.2	12.47
r	+20	19.2668	23.5948	854.3	-4.89
	+15	18.9424	22.5859	866.1	-3.58
	+10	18.6820	21.8379	877.2	-2.34

	-10	17.9093	19.8848	918.0	2.20
	-15	17.7527	19.5304	927.5	3.26
	-20	17.6056	19.2078	936.8	4.29
<i>a</i>	+50	14.1455	16.2044	1040.5	15.84
	+30	15.5167	17.7068	993.7	10.63
	+10	17.2218	19.5793	934.2	4.00
	-10	19.4706	22.0669	856.8	-4.61
	-30	22.7142	25.7161	751.3	-16.36
	-50	28.1480	32.0997	594.5	-33.81
<i>b</i>	+20	17.0250	18.3033	922.2	2.67
	+15	17.3015	18.8071	918.2	2.22
	+10	17.5966	19.3676	912.9	1.63
	-10	19.0636	22.5984	876.7	-2.40
	-15	19.5583	23.9140	862.7	-3.96
	-20	20.1812	25.8140	845.8	-5.84
<i>S</i>	+20	19.7056	24.3198	1024.8	14.09
	+15	19.3262	23.2679	995.5	10.83
	+10	18.9641	22.3440	964.7	7.40
	-10	17.5472	19.2765	825.4	-8.11
	-15	17.1807	18.5907	786.4	-12.45
	-20	16.8028	17.9196	745.7	-16.98

From Table 1, the following points are noted:

- (i) It is seen that the percentage change in the optimal cost is almost equal for both positive and negative changes of all the parameters except c_1 , c_2 , α , k , a and b
- (ii) It is observed that the model is more sensitive for a negative change than an equal positive change in these parameters.
- (iii) The optimal cost increases (decreases) with the increase (decrease) in the value of all the parameters except k and r . This trend is reversed for the parameter k and r .
- (iv) Model is highly sensitive to changes in c_1 , k , a & S . Model is moderately sensitive to changes in c_2 , α & b . It has low sensitivity to c' , c_3 , c_4 & λ .
- (v) From the above points, it is clear that much care is to be taken to estimate c_1 , k , a & S .

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